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FREE VIBRATION OF ISOTROPIC, ORTHOTROPIC, AND MULTILAYER PLATES BASED ON HIGHER ORDER REFINED THEORIES

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Laminated composite plates are being increasingly used in the aeronautical and aerospace industry as well as in other fields of modern technology. To use them effeciently a good understanding of their structural and dynamical behaviour is needed. The *Classical Laminate Plate Theory* [1] which ignores the effect of transverse shear deformation becomes inadequate for the analysis of multilayer composites. The first order theories (FSDTs) based on Reissner [2] and Mindlin [3] assume linear in-plane stresses and displacements, respectively, through the laminate thickness. Since FSDTs account for layerwise constant states of transverse shear stress, shear correction coefficients are needed to rectify the unrealistic variation of the shear strain/stress through the thickness and which ultimately define the shear strain energy. In order to overcome the limitations of FSDTs, higher order shear deformation theories (HSDTs) that involve higher order terms in Taylor's expansions of the displacement in the thickness co-ordinate were developed. Hildebrand *et al.* [4] were the first to introduce this approach to derive improved theories of plates and shells. Kant [5] was the first to derive the complete set of variationally consistent governing equations for the flexure of a symmetrically laminated composite plate incorporating both distortion of transverse normals and effects of transverse normal stress/strain by utilizing the complete three-dimensional generalized Hooke's law and presented results for isotropic plate only. Later Mallikarjuna [6], Mallikarjuna and Kant [7] and Kant and Mallikarjuna [8, 9] presented a set of higher order refined theories and presented formulations and solutions for the free vibration analysis of general laminated composite and sandwich plate problems based on finite element methods. *In this investigation, analytical solutions for the free vibration analysis of laminated composite and sandwich plates based on two higher order refined theories already developed by the first author for which analytical formulations and solutions were not reported earlier in the literature are presented.* After establishing the accuracy of the present results with three-dimensional elasticity solutions for isotropic, orthotropic and composite plates, benchmark results and comparison of solutions using various theories are presented for multilayer sandwich plates.

The displacement models under various theories considered in the present investigations are listed below [10–14]:

Model—1 (Kant and Manjunatha, 1988):

$$u(x, y, z) = u_0(x, y) + z\theta_x(x, y) + z^2 u_0^*(x, y) + z^3\theta_x^*(x, y),$$

$$v(x, y, z) = v_0(x, y) + z\theta_y(x, y) + z^2 v_0^*(x, y) + z^3\theta_y^*(x, y),$$

$$w(x, y, z) = w_0(x, y) + z\theta_z(x, y) + z^2 w_0^*(x, y) + z^3\theta_z^*(x, y). \tag{1}$$

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Model—2 (Pandya and Kant, 1988):

$$u(x, y, z) = u_0(x, y) + z\theta_x(x, y) + z^2 u_0^*(x, y) + z^3 \theta_x^*(x, y),$$

$$v(x, y, z) = v_0(x, y) + z\theta_y(x, y) + z^2 v_0^*(x, y) + z^3 \theta_y^*(x, y),$$

$$w(x, y, z) = w_0(x, y). \tag{2}$$

Though the above two theories were already reported in the literature and numerical results were presented using finite element formulations, analytical formulations and solutions have been obtained for the first time in this investigation and so the results obtained using the above two theories are referred to as *present* in all the tables. In addition to the above, the following higher order theories and the first order theory developed by other investigators and reported in the literature for the analysis of laminated composite and sandwich plates are also considered for the evaluation. Analytical formulations and numerical results of these are also being presented here with a view to have all the results on a common platform.

Model—3 (Reddy, 1984):

$$u(x, y, z) = u_0(x, y) + z\left[\theta_x(x, y) - \frac{4}{3}\left(\frac{z}{h}\right)^2 \left\{\theta_x(x, y) + \frac{\partial w_0}{\partial x}\right\}\right],$$

$$v(x, y, z) = v_0(x, y) + z\left[\theta_y(x, y) - \frac{4}{3}\left(\frac{z}{h}\right)^2 \left\{\theta_y(x, y) + \frac{\partial w_0}{\partial y}\right\}\right],$$

$$w(x, y, z) = w_0(x, y). \tag{3}$$

Model—4 (Senthilnathan *et al.*, 1987):

$$u(x, y, z) = u_0(x, y) - z\frac{\partial w_0^b}{\partial x} - \frac{4z^3}{3h^2}\frac{\partial w_0^s}{\partial x},$$

$$v(x, y, z) = v_0(x, y) - z\frac{\partial w_0^b}{\partial y} - \frac{4z^3}{3h^2}\frac{\partial w_0^s}{\partial y},$$

$$w(x, y, z) = w_0^b(x, y) + w_0^s(x, y). \tag{4}$$

Model—5 (Whitney and Pagano, 1970):

$$u(x, y, z) = u_0(x, y) + z\theta_x(x, y)$$

$$v(x, y, z) = v_0(x, y) + z\theta_y(x, y)$$

$$w(x, y, z) = w_0(x, y). \tag{5}$$

The definitions of parameters in equations (1)–(5) are not being repeated here for the sake of brevity. A simply (diaphragm) supported square plate is considered throughout as a test problem. The composite structures studied in this investigation are fibre-reinforced laminated composite and sandwich plates. The equations of motion of all the displacement models are derived using Hamilton's principle. Solutions are obtained in closed-form using Navier's solution technique and by solving the eigenvalue problem.

The non-dimensionalized natural frequencies $\bar{\omega}$ of general rectangular isotropic, orthotropic, composite and sandwich plates are considered for comparison. Natural frequencies with the percentage error with respect to three-dimensional elasticity solutions [15] for a thick square isotropic plate ($v = 0{\cdot}3$) are given in Table 1. A shear correction factor of 5/6 is used in computing results using Whitney–Pagano's theory. Comparison of results show that the theory of Kant–Manjunatha which takes into account both the transverse shear and transverse normal deformation, predicts the natural frequencies with the same degree of accuracy as that of (3-D) three-dimensional elasticity solutions at lower as well as at higher modes. In all the other theories where the transverse normal deformation is neglected the error is quite considerable both at lower and higher modes especially when plates are thick.

Results obtained for a single-layer square orthotropic plate are given in Table 2. The following elastic constants are used [16]:

$$C_{11} = 23{\cdot}2 \times 10^6 \text{ psi (160 GPa)}, \quad C_{12} = 5{\cdot}41 \times 10^6 \text{ psi (37·3 GPa)},$$

$$C_{13} = 0{\cdot}25 \times 10^6 \text{ psi (1·72 GPa)}, \quad C_{22} = 12{\cdot}6 \times 10^6 \text{ psi (86·87 GPa)},$$

$$C_{23} = 2{\cdot}28 \times 10^6 \text{ psi (15·72 GPa)}, \quad C_{33} = 12{\cdot}3 \times 10^6 \text{ psi (84·81 GPa)},$$

$$C_{44} = 6{\cdot}10 \times 10^6 \text{ psi (42·06 GPa)}, \quad C_{55} = 6{\cdot}19 \times 10^6 \text{ psi (42·68 GPa)},$$

$$C_{66} = 3{\cdot}71 \times 10^6 \text{ psi (25·58 GPa)}.$$

Comparison of results indicates that the percentage of error with respect to three-dimensional elasticity solutions [16] is almost nil in the case of Kant–Manjunatha theory whereas in the case of other models the error is quite significant. The non-dimensionalized natural frequencies of three-, five- and nine-layer symmetric cross-ply laminate with layers of equal thickness are given in Table 3.

The orthotropic material properties of individual layers in all the above laminates considered are $E_1/E_2 = \text{open}$, $E_2 = E_3$, $G_{12} = G_{13} = 0{\cdot}6E_2$, $G_{23} = 0{\cdot}5E_2$, $v_{12} = v_{13} = v_{23} = 0{\cdot}25$. Three-dimensional elasticity solutions given by Noor [17] is considered for comparison. For a three-layer symmetric laminate where the effect of transverse deformation is more pronounced the percentage error with respect to 3-D elasticity solutions is less in Kant–Manjunatha theory compared to other theories for all ranges of E_1/E_2. The percentage error in all the theories increases with the increase in the degree of anisotropy. For the range of E_1/E_2 from 10 to 40, the percentage error in predicting the natural frequencies using the theory of Senthilnathan *et al.* is very high compared to other theories, the maximum being 9·48 per cent at $E_1/E_2 = 40$. As the number of layer increases, the error in the results obtained using the different theories decreases significantly.

The results of a five-layer sandwich plates with antisymmetric cross-ply faces are shown in Tables 4 and 5. Both thin and thick laminates are considered. The following material properties are used for the face sheets and the core [18]:

TABLE 1

Natural frequencies $\bar{\omega} = \omega h\sqrt{\rho/G}$ of an isotropic plate with $\nu = 0{\cdot}3$, $a/h = 10$ and $a/b = 1$

m	n	Present Model-1	Present Model-2	Reddy[†]	Senthilnathan *et al.*[†]	Whitney–Pagano[†]	3-D elasticity
1	1	0·0932 (0·0)[‡]	0·0930 (− 0·21)	0·0930 (− 0·21)	0·0930 (− 0·21)	0·0930 (− 0·21)	0·0932
1	2	0·2226 (0·0)	0·2220 (− 0·27)	0·2220 (− 0·27)	0·2220 (− 0·27)	0·2220 (− 0·27)	0·2226
2	2	0·3421 (0·0)	0·3406 (− 0·44)	0·3406 (− 0·44)	0·3406 (− 0·44)	0·3406 (− 0·44)	0·3421
1	3	0·4172 (0·02)	0·4151 (− 0·48)	0·4151 (− 0·48)	0·4150 (− 0·50)	0·4149 (− 0·53)	0·4171
2	3	0·5240 (0·02)	0·5208 (− 0·59)	0·5208 (− 0·59)	0·5208 (− 0·59)	0·5206 (− 0·63)	0·5239
1	4	0·6573	0·6525	0·6525	0·6524	0·6520	—
3	3	0·6892 (0·04)	0·6839 (− 0·73)	0·6839 (− 0·73)	0·6839 (− 0·73)	0·6834 (− 0·80)	0·6889
2	4	0·7515 (0·05)	0·7453 (− 0·77)	0·7454 (− 0·76)	0·7453 (− 0·77)	0·7447 (− 0·85)	0·7511
3	4	0·8992	0·8908	0·8908	0·8908	0·8896	—
1	5	0·9275 (0·08)	0·9186 (− 0·88)	0·9187 (− 0·87)	0·9186 (− 0·88)	0·9174 (− 1·01)	0·9268
2	5	1·0102	1·0000	1·0000	1·0000	0·9984	—
4	4	1·0899 (0·0)	1·0784 (− 0·96)	1·0784 (− 0·96)	1·0784 (− 0·96)	1·0764 (− 0·96)	1·0889
3	5	1·1416	1·1291	1·1291	1·1292	1·1269	—

[†] Results using these theories are computed independently and are found to be the same as the results reported earlier in various references.
[‡] Numbers in parentheses are the percentage error with respect to 3-D elasticity values.

TABLE 2

Natural frequencies $\bar{\omega} = \omega h \sqrt{\rho / c_{11}}$ *of a single-layer square orthotropic plate with* $a/h = 10$ *and* $c_{11} = 23{\cdot}2 \times 10^6$ *psi* (160 *GPa*)

m	n	Present Model-1	Present Model-2	Reddy[†]	Senthilnathan *et al.*[†]	Whitney–Pagano[†]	3-D elasticity
1	1	0·0474 (0·0)[‡]	0·0476 (0·42)	0·0476 (0·42)	0·0478 (0·84)	0·0476 (0·42)	0·0474
1	2	0·1033 (0·0)	0·1041 (0·77)	0·1041 (0·77)	0·1049 (1·55)	0·1041 (0·77)	0·1033
2	1	0·1188 (0·0)	0·1189 (0·08)	0·1189 (0·08)	0·1198 (0·84)	0·1188 (0·0)	0·1188
2	2	0·1694 (0·0)	0·1698 (0·24)	0·1698 (0·24)	0·1726 (1·89)	0·1698 (0·24)	0·1694
1	3	0·1888 (0·0)	0·1906 (0·95)	0·1906 (0·95)	0·1919 (1·64)	0·1905 (0·90)	0·1888
3	1	0·2181 (0·05)	0·2181 (0·05)	0·2181 (0·05)	0·2197 (0·78)	0·2178 (− 0·09)	0·2180
2	3	0·2476 (0·04)	0·2487 (0·48)	0·2487 (0·48)	0·2533 (2·34)	0·2485 (0·40)	0·2475
3	2	0·2625 (0·04)	0·2626 (0·08)	0·2626 (0·08)	0·2677 (2·02)	0·2623 (− 0·04)	0·2624
1	4	0·2969 (0·0)	0·2995 (0·88)	0·2995 (0·88)	0·3012 (1·45)	0·2994 (0·84)	0·2969
4	1	0·3319 (0·0)	0·3319 (0·0)	0·3320 (0·03)	0·3340 (0·63)	0·3340 (0·63)	0·3319
3	3	0·3320 (0·0)	0·3326 (0·18)	0·3326 (0·18)	0·3414 (2.83)	0·3321 (0·03)	0·3320
2	4	0·3476 (0·0)	0·3495 (0·55)	0·3495 (0·55)	0·3558 (2·36)	0·3491 (0·43)	0·3476
4	2	0·3707 (0·0)	0·3707 (0·0)	0·3708 (0·03)	0·3775 (1·83)	0·3698 (− 0·24)	0·3707

Note: For †, ‡ see footnotek to Table 1.

TABLE 3

Non-dimensionalized fundamental frequencies $\bar{\omega} = (\omega b^2/h)\sqrt{\rho/E_2}$ *for a simply supported cross-ply square laminated plates with* $a/h = 5$

Lamination and No. of layers	Source	E_1/E_2				
		3	10	20	30	40
$(0/9\bar{0})_s$	3-D elasticity	6·6185	8·2103	9·5603	10·2723	10·7515
	Present (Model–1)	6·5712 (– 0·71)[†]	8·1696 (– 0·50)	9·2513 (– 3·23)	9·8595 (– 4·02)	10·2686 (– 4·49)
	Present (Model–2)	6·5523 (– 1·00)	8·1508 (– 0·72)	9·2335 (– 3·42)	9·8428 (– 4·18)	10·2529 (– 4·64)
	Reddy[‡]	6·5527 (– 0·99)	8·1510 (– 0·72)	9·2348 (– 3·40)	9·8474 (– 4·14)	10·2631 (– 4·54)
	Senthilnathan *et al.*,[‡]	6·6003 (– 0·27)	8·5731 (4·41)	10·1516 (6·18)	11·1132 (8·19)	11·7710 (9·48)
	Whitney–Pagano[‡]	6·5630 (– 0·84)	8·1847 (– 0·31)	9·2774 (– 2·90)	9·8851 (– 3·77)	10·2894 (– 4·30)
$(0/90/\bar{0})_s$	3-D elasticity	6·6468	8·5223	9·948	10·785	11·3435
	Present (Model–1)	6·6033 (– 0·65)	8·4382 (– 0·99)	9·8246 (– 1·24)	10·6437 (– 1·31)	11·1957 (– 1·30)
	Present (Model–2)	6·5842 (– 0·94)	8·4186 (– 1·22)	9·8062 (– 1·43)	10·6270 (– 1·46)	11·1806 (– 1·44)
	Reddy[‡]	6·5850 (– 0·93)	8·4308 (– 1·07)	9·8413 (– 1·07)	10·6856 (– 0·92)	11·2617 (– 0·72)
	Senthilnathan *et al.*,[‡]	6·6003 (– 0·70)	8·5731 (0·60)	10·1515 (2·05)	11·1132 (3·04)	11·7710 (3·77)
	Whitney–Pagano[‡]	6·5844 (– 0·94)	8·4201 (– 1·20)	9·8265 (– 1·22)	10·6785 (– 0·98)	11·2671 (– 0·67)
$(0/90/0/90/\bar{0})_s$	3-D elasticity	6·66	8·608	10·1368	11·0525	11·6698
	Present (Model–1)	6·6143 (– 0·69)	8·5422 (– 0·76)	10·0546 (– 0·81)	10·9643 (– 0·80)	11·5811 (– 0·76)
	Present (Model–2)	6·5952 (– 0·97)	8·5228 (– 0·99)	10·0368 (– 0·99)	10·9487 (– 0·94)	11·5676 (– 0·88)
	Reddy[‡]	6·5959 (– 0·96)	8·5311 (– 0·89)	10·0598 (– 0·76)	10·9866 (– 0·60)	11·6198 (– 0·43)
	Senthilnathan *et al.*,[‡]	6·6003 (– 0·90)	8·5731 (– 0·41)	10·1516 (0·15)	11·1132 (0·55)	11·7710 (0·87)
	Whitney–Pagano[‡]	6·5940 (– 0·99)	8·5196 (– 1·03)	10·0366 (– 0·99)	10·9544 (– 0·89)	11·5787 (– 0·78)

[†] Numbers in parentheses are the percentage error with respect to 3-D elasticity values.
[‡] Results using these theories are computed independently and are found to be the same as the results repored earlier in various references.

TABLE 4

Natural frequencies $\bar{\omega} = (\omega b^2/h)\sqrt{(\rho/E_2)_f}$ *of unsymmetric* (0/90/*core*/0/90) *sandwich plate with* $a/h = 10$, $a/b = 1$ *and* $t_c = t_f = 10$

m	n	Present Model-1	Present Model-2	Reddy†	Senthilnathan *et al.*†	Whitney–Pagano†
			Considering G_{13} and G_{23} of stiff layers			
1	1	4·8594	4·8519	7·0473	7·0473	13·8694
1	2	8·0187	7·9965	11·9087	11·9624	30·6432
1	3	11·7381	11·6809	17·3211	17·3698	50·9389
2	2	10·2966	10·2550	15·2897	15·2897	41·5577
2	3	13·4706	13·3889	19·8121	19·8325	58·3636
3	3	16·1320	16·0039	23·5067	23·5067	71·3722
			Neglecting G_{13} and G_{23} of stiff layers			
1	1	1·5617	1·5602	1·8237	1·8237	1·4473
1	2	2·4938	2·4921	3·0801	3·0808	2·2941
1	3	3·5424	3·5409	4·8053	4·8058	3·2469
2	2	3·1623	3·1604	4·0417	4·0417	2·9032
2	3	4·0411	4·0394	5·5754	5·5756	3·7024
3	3	4·7599	4·7582	6·9098	6·9098	4·3573

Note: for † see footnote to Table 1.

Face sheets (Graphite-epoxy T300/934):

$$E_1 = 19 \times 10^6 \text{ psi } (131 \text{ GPa}), \quad E_2 = 1{\cdot}5 \times 10^6 \text{ psi } (10{\cdot}34 \text{ GPa}),$$

$$E_2 = E_3,$$

$$G_{12} = 1 \times 10^6 \text{ psi } (6{\cdot}895 \text{ GPa}), \quad G_{13} = 0{\cdot}90 \times 10^6 \text{ psi } (6{\cdot}205 \text{ GPa}),$$

$$G_{23} = 1 \times 10^6 \text{ psi } (6{\cdot}895 \text{ GPa}),$$

$$\nu_{12} = 0{\cdot}22, \qquad \nu_{13} = 0{\cdot}22, \qquad \nu_{23} = 0{\cdot}49$$

$$\rho = 0{\cdot}057 \text{ 1b/in}^3 \ (1627 \text{ kg/m}^3).$$

Core properties (isotropic):

$$E_1 = E_2 = E_3 = 2G = 1000 \text{ psi } (6{\cdot}89 \times 10^{-3} \text{ GPa}),$$

$$G_{12} = G_{13} = G_{23} = 500 \text{ psi } (3{\cdot}45 \times 10^{-3} \text{ GPa}),$$

$$\nu_{12} = \nu_{13} = \nu_{23} = 0,$$

$$\rho = 0{\cdot}3403 \times 10^{-2} \text{ 1b/in}^3 \ (97 \text{ kg/m}^3).$$

The effect of transverse shear rigidities of stiff layers and side-to-thickness ratio on the natural frequencies are studied. It is seen that both for thick and thin plates the results of Kant–Manjunatha and Pandya–Kant are in good agreement. For thick plate with the transverse shear moduli (G_{23} and G_{13}) of stiff layers included, the difference in predicting the natural frequencies between the theory of Kant–Manjunatha and the theories of Reddy and Senthilnathan *et al.* increases with the increasing mode number. The first order theory

TABLE 5

Natural frequencies $\bar{\omega} = (\omega b^2/h)\sqrt{(\rho/E_2)_f}$ *of unsymmetric* (0/90/*core*/0/90) *sandwich plate with* $a/h = 100$, $a/b = 1$ *and* $t_c = t_f = 10$

m	n	Present Model-1	Present Model-2	Reddy†	Senthilnathan *et al.*†	Whitney–Pagano†
			Considering G_{13} and G_{23} of stiff layers			
1	1	15·5093	15·4646	15·9521	15·9521	16·2175
1	2	39·0293	38·9232	42·2271	42·3708	44·7072
1	3	72·7572	72·5925	83·9982	84·4251	94·9097
2	2	54·7618	54·6330	60·1272	60·1272	64·5044
2	3	83·4412	83·2699	96·3132	96·7159	108·9049
3	3	105·3781	105·1807	124·2047	124·2047	143·7969
			Neglecting G_{13} and G_{23} of stiff layers			
1	1	11·2025	11·1855	11·9838	11·9838	10·8311
1	2	21·2525	21·2333	23·5260	23·7778	20·2688
1	3	32·2823	32·2630	36·3449	36·6482	30·5730
2	2	27·9082	27·8879	31·1132	31·1132	26·5301
2	3	37·0027	36·9802	41·6740	41·8358	35·0181
3	3	44·2389	44·2121	50·0225	50·0225	41·7761

Note: for † see footnote to Table 1.

very much overestimates the frequency values at lower as well as at higher modes From the results of natural frequencies of thin laminate shown in Table 5, it can be concluded that the effect of transverse shear moduli of stiff layers is more pronounced in thick laminates than for thin laminates. The idea behind this entire investigation is to bring out clearly the accuracy of the various shear deformation theories in predicting the natural frequencies so that the claims made by various investigators regarding the supremacy of their models are put to rest.

REFERENCES

1. E. REISSNER and Y. STAVSKY 1961 *American Society of Mechanical Engineers Journal of Applied Mechanics* **28**, 402–408. Bending and stretching of certain types of heterogeneous aelotropic elastic plates.
2. E. REISSNER 1945 *American Society of Mechanical Engineers Journal of Applied Mechanics* **12**, 69–77. The effect of transverse shear deformation on the bending of elastic plates.
3. R. D. MINDLIN 1951 *American Society of Mechanical Engineers Journal of Applied Mechanics* **18**, 31–38. Influence of rotary inertia and shear on flexural motions of isotropic, elastic plates.
4. F. B. HILDEBRAND, E. REISSNER and G. B. THOMAS 1949 *NACA TN*-1833. Note on the foundations of the theory of small displacements of orthotropic shells.
5. T. KANT 1982 *Computer Methods in Applied Mechanics and Engineering* **31**, 1–18. Numerical analysis of thick plates.
6. MALLIKARJUNA 1988 *Ph.D. thesis, Indian Institute of Technology Bombay, Mumbai, India*. Refined theories with C° finite elements for free vibration and transient dynamics of anisotropic composite and sandwich plates.
7. MALLIKARJUNA and T. KANT 1989 *International Journal for Numerical Methods in Engineering* **28**, 1875–1889. Free vibration of symmetrically laminated plates using a higher-order theory with finite element technique.
8. T. KANT and MALLIKARJUNA 1989 *Computer and Structures* **32**, 1125–1132. A higher-order theory for free vibration of unsymmetrically laminated composite and sandwich plates—finite element evaluations.

9. T. Kant and Mallikarjuna 1989 *Journal of Sound and Vibration* **134**, 1–16. Vibration of unsymmetrically laminated plates analysed by using a higher-order theory with a C° finite element formulation.
10. T. Kant and B. S. Manjunatha 1988 *Engineering Computations* **5**, 300–308. An unsymmetric FRC laminate C° finite element model with 12 degrees of freedom per node.
11. B. N. Pandya and T. Kant 1988 *Composite Science and Technology* **32**, 137–155. Finite element stress analysis of laminated composite plates using higher-order displacement model.
12. J. N. Reddy 1984 *American Society of Mechanical Engineers Journal of Applied Mechanics* **51**, 745–752. A simple higher-order theory for laminated composite plates.
13. N. R. Senthilnathan, K. H. Lim, K. H. Lee and S. T. Chow 1987 *American Institute of Aeronautics and Astronautics Journal* **25**, 1268–1271. Buckling of shear-deformable plates.
14. J. M. Whitney and N. J. Pagano 1970 *American Society of Mechanical Engineers Journal of Applied Mechanics* **37**, 1031–1036. Shear deformation in heterogeneous anisotropic plates.
15. S. Srinivas, C. V. Joga Rao and A. K. Rao 1970 *Journal of Sound and Vibration* **12**, 187–199. An exact analysis for vibration of simply supported homogeneous and laminated thick rectangular plates.
16. S. Srinivas and A. K. Rao 1970 *International Journal of Solids and Structures* **6**, 1463–1481. Bending vibration and buckling of simply supported thick orthotropic plates and laminates.
17. A. K. Noor 1973 *American Institute of Aeronautics and Astronautics Journal* **11**, 1038–1039. Free vibrations of multilayered composite plates.
18. J. N. Reddy 1996 *Mechanics of Laminated Composite Plates, Theory and Analysis*. Boca Raton, FL, U.S.A.: CRC Press, Inc.